Department of Mathematics and Statistics

2015 PhD Projects

The Open University

Project title	Renormalization for asymmetric dynamical systems
Principal supervisor	Ben Mestel
Second supervisor	Ian Short
Discipline	Pure mathematics
Research area/keywords	dynamical systems, complex analysis, renormalization, functional equations
Suitable for	Full time students

Project background and description

The study via renormalization of universal properties of chaotic transitions (such as the celebrated Feigenbaum period-doubling cascade for unimodal maps) leads to nonlinear functional equations that are particularly hard to analyse. Following pioneering work of Henri Epstein, Dennis Sullivan, Curt McMullen and Mikhail Lyubich, amongst others, these equations may be elegantly studied using advanced complex analysis.

Asymmetric unimodal maps can arise as Poincaré maps of discontinuous continuous-time systems and undergo period-doubling cascades analogously to symmetric unimodal maps. They also exhibit universality in their chaotic transitions, distinct from the symmetric case.

This project will extend the theory for symmetric unimodal maps to asymmetric unimodal maps thereby establishing the universality of asymmetric period-doubling cascades for even degree of criticality.

This project requires knowledge of real and complex analysis and of dynamical systems theory. You will work closely with the principal supervisor, studying in detail the work of McMullen and Lyubich, and developing their ideas to the asymmetric case.

Background reading/references

- Mikhail Lyubich, The Quadratic Family as a Qualitatively Solvable Model of Chaos, *Notices of the AMS* **47** (9) (2000) http://www.math.sunysb.edu/~mlyubich/papers/Notices.pdf
- B.D. Mestel and A H. Osbaldestin, Feigenbaum theory for unimodal maps with asymmetric critical point, *J. Phys.* **31** (1998), 3287–3296.
- B. D. Mestel and A.H. Osbaldestin, Feigenbaum theory for unimodal maps with asymmetric critical point: rigorous results, *Commun. Math. Phys.* 197 (1998), 211–228.
- Curtis T. McMullen, Renormalization and 3-Manifolds Which Fiber over the Circle, *Annals of Mathematics Studies*, Princeton University Press, 1996