Department of Mathematics and Statistics

2015 PhD Projects

Project title	Extremal vertex-transitive graphs and embeddings
Principal supervisor	Jozef Siran
Second supervisor	Robert Brignall
Discipline	Pure mathematics
Research area/keywords	Vertex-transitive graph; Cayley graph; Regular map
Suitable for	Full time or part time students

Project background and description

The study of symmetry in graphs and maps has a long and rich history, is closely related to other mathematical disciplines such as group theory, finite geometry, low-dimensional topology and the theory of Riemann surfaces, and has numerous applications in theoretical computer science.

The aim of this project is to study extremal vertex-transitive graphs with respect to diameter (or girth) on the one hand, and with respect to embeddability on surfaces on the other hand. The problem of determination of the largest order of a graph of a given maximum degree and diameter (known as the degree-diameter problem) has attracted considerable attention over the past 50 years, with hundreds of papers published on the topic. The situation is similar with the (in some sense, dual) problem of determining the smallest order of a regular graph of a given degree and girth (length of the shortest cycle). Both problems have been considered in their restricted versions to vertex-transitive graphs and this study is closely related to group theory and finite geometry, with applications in network design in theoretical computer science. As regards embeddings (also called maps) on orientable surfaces, the highest 'level of symmetry' is obtained when the automorphism group of an embedding is transitive (and hence regular) on the vertex-edge incident pairs. Such maps are called regular; they are generalisations of Platonic solids to arbitrary surfaces and have close ties to group theory, low-dimensional topology and the theory of Riemann surfaces.

This PhD project will contribute to the above research areas by explicit constructions of vertex-transitive and Cayley graphs in the degree-diameter and degree-girth problem, and constructions of new classes of regular maps.

Background reading/references

- M. Miller and J. Siran, Moore graphs and beyond – a survey of the degree-diameter problem. Electronic. J. Combin., Dynamic Survey DS14, 2013, 91 pages. http://www.combinatorics.org/issue/view/Surveys
- J. Siran, How symmetric can maps on surfaces be? In: Surveys in Combinatorics (London Mathematical Society Lecture Note Series 409), Cambridge University Press, 2013, 161–238.