Department of Mathematics and Statistics

2015 PhD Projects

The Open University

Project title	Counting permutations using generalised grid classes
Principal supervisor	Robert Brignall
Second supervisor	Jozef Siran
Discipline	Pure mathematics
Research area/keywords	permutation, combinatorics, enumeration, pattern
Suitable for	Full time students, or strong part time students able to devote more than 50% of their time to research

Project background and description

The study of *permutation patterns* looks at permutations not as objects from group theory, but as combinatorial objects (more like graph theory). Writing permutations in one-line notation (rather than, e.g., cycle notation), a permutation π is said to be *contained* in another σ if there is a subsequence of σ which has the same relative ordering as π. For example, $\pi = 132$ is contained in $\sigma = 24153$ because the three entries $\sigma(1)\sigma(4)\sigma(5) = 253$ are in the same order (i.e. form the same 'pattern') as $\pi = 132$.

This notion of containment forms a partial order on the set of all permutations. One can then define classes of permutations which *avoid* a specific permutation or set of permutations, for example the notation $\mathrm{Av}(132)$ refers to the set (or class) of all permutations which do not contain 132. A typical question here is to count how many permutations there are of each length n: for $\mathrm{Av}(132)$ it turns out that this sequence is counted by the Catalan numbers.

In recent years, a rich structure theory has been emerging, and among other applications this can be used to count the permutations in a class. One particularly potent tool has been the notion of *grid classes* – see, for example, [1] – this has been used to great effect to count the permutations in many more classes, for example several of those listed in [2].

The notion of *generalized grid classes* is less developed, and to-date has not been used much to solve specific counting problems. The starting point of this project is to use generalized grid classes to count the permutations in several more classes (some of those listed in [3] but not yet counted have an appropriate form), and in so doing develop techniques and tools which other researchers can use. The precise programme of research will depend on the initial progress and interests of the successful applicant.

Background reading/references

[1] Albert, M.H., Atkinson, M.D., and Brignall, R., The enumeration of three pattern classes using monotone grid classes. *Electronic Journal of Combinatorics* **19**(3) (2012), #P20 (34pp). http://users.mct.open.ac.uk/rb8599/papers/ThreeEnumerations.pdf

[2] Enumerations of Specific permutation classes, *Wikipedia*, Accessed 4th February 2015. http://en.wikipedia.org/wiki/Enumerations_of_specific_permutation_classes